Introduction to Engineering Economic Analysis

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Topics

- Introduction to engineering economic analysis
- Time value of money
- The principle of interest,
- Simple interest rates
- Compounding interest rates
- The principle of discounting.

Engineering Economics

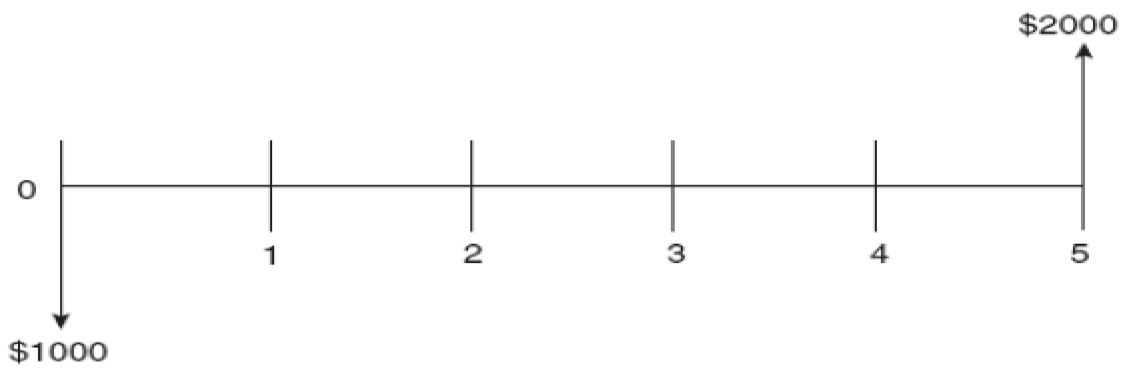
- Engineering economics is the application of economic techniques to the evaluation of design and engineering alternatives
- The role of engineering economics is to assess the engineering of a give project with the justification for their decisions based on engineering standpoint

Engineering Economics

- Time value of money is defined as the time-dependent value of money stemming from changes in the purchasing power of money and from the real earning potential of alternative investments over time

Cash Flow representation

- It is difficult to solve a problem if you can not see it.
- The easiest way to approach problems in economic analysis is to draw a picture:
  - Time interval divided in equal periods
  - All cash outflows
  - All cash inflows



Introduction

- If you win a prize of $100,000 and you were asked to choose to have this money today or to have it a year a year later, what would you choose?
- Off course you will choose to have the money now for the following reasons:
  - ***Growth of money:*** extra money can be earned by good investing
  - ***Inflation:*** the purchasing power of the money a year later will decline
  - ***Risk:*** there is a chance of being unable to receive the money due to unexpected reasons

Introduction

- Assume you have invested \$100,000 now, the money grows to \$125,000 after first year and then to \$150,000 after two years.
  - Although the money grows in two years from 100,000 to 150,000, the investor suffers at the same time a loss due to the inflation of money in this period.
  - If the inflation rate is higher than the bank interest, it would result in a net loss for depositing money in a bank

Interest

- Interest has two types:

1. Simple Interest does not pay interest on the previous period

2. Compound Interest pay interest on the previous period

Simple Interest

- **Simple Interest**

$$I=P(i)n$$

where:

- I = Interest ,
- P = Principal,
- n = No of years ,
- i = Interest rate per year

Simple Interest

If you have $100 and will be invested using simple interest with 10%

- After 1 year, the $100 will be $110
- After 2 years, the $100 will be $120
- After 5 years, the $100 will be $150

- $F_1 = P + P \times i = P\ (1+i)$
- $F_2 = P + P \times i + P \times i = P\ (1+2 \times i)$

$$\mathbf{F_N = P\ (1+N \times \mathit{i})}$$

Compound Interest

If you have \$100 and will be invested using compound interest with 10%

- After 1 year, the \$100 will be \$110
- After 2 years, the \$100 will be \$120
- After 5 years, the \$100 will be \$161

- $F_1 = P + P \times i = P\,(1+i)$
- $F_2 = P \times (1+i) \times (1+i)$

$$\mathbf{F_N = P\,(1+\mathit{i})^N}$$

Discount interest

- The inverse of compounding is determining a present amount which will yield a specified future sum.
- The equation for discounting is found by:

- $\mathbf{P_N = F\,(1+\mathit{i})^{-N}}$

Series Compound factor

- Given a series of regular payment, what will they be worth at some future time
- A = the amount of a regular end-of-period payment
- Each payment A, is compounded for a different period of time
- $\boldsymbol{F = A\left[\frac{(1+i)^{N}-1}{i}\right]}$
- $\boldsymbol{F = A\,(F/A, i, N)}$

Sinking Fund Factor

The process corresponding to the inverse of series compounding is referred to as a sinking fund; that is, what size regular series payment are necessary to acquire a given future amount?

$$A = F\left[\frac{i}{(1+i)^N - 1}\right]$$

- $A = F\,(A/F, i, N)$

Series Present Worth

$$P = A \frac{[(1+i)^N - 1]}{[i(1+i)^N]}$$

$$P = A\,(P/A, i, N)$$

Factor Name	Converts	Symbol	Formula
Single payment compound interest	P to F	(F/P,i%,n)	$(1 + i)^n$
Present Worth	F to P	(P/F,i%,n)	$1/(1 + i)^n$
Uniform series Sinking Fund	F to A	(A/F,i%,n)	$i/[(1 + i)^n - 1]$
Capital Recovery	P to A	(A/P,i%,n)	$[i(1 + i)^n]/[(1 + i)^n - 1]$
Compound amount	A to F	(F/A,i%,n)	$[(1 + i)^n - 1]/i$
Equal series present worth	A to P	P/A,i%,n)	$[(1 + i)^n - 1]/[i(1 + i)^n]$

INTEREST RATE = 5.00%

N	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	N
1	0.9524	0.9524	0.0000	1.0500	1.0000	1.0500	1.0000	1
2	0.9070	1.8594	0.9070	1.1025	2.0500	0.5378	0.4878	2
3	0.8638	2.7232	2.6347	1.1576	3.1525	0.3672	0.3172	3
4	0.8227	3.5460	5.1028	1.2155	4.3101	0.2820	0.2320	4
5	0.7835	4.3295	8.2369	1.2763	5.5256	0.2310	0.1810	5
6	0.7462	5.0757	11.9680	1.3401	6.8019	0.1970	0.1470	6
7	0.7107	5.7864	16.2321	1.4071	8.1420	0.1728	0.1228	7
8	0.6768	6.4632	20.9700	1.4775	9.5491	0.1547	0.1047	8
9	0.6446	7.1078	26.1268	1.5513	11.0266	0.1407	0.0907	9
10	0.6139	7.7217	31.6520	1.6289	12.5779	0.1295	0.0795	10
11	0.5847	8.3064	37.4988	1.7103	14.2068	0.1204	0.0704	11
12	0.5568	8.8633	43.6241	1.7959	15.9171	0.1128	0.0628	12
13	0.5303	9.3936	49.9879	1.8856	17.7130	0.1065	0.0565	13
14	0.5051	9.8986	56.5538	1.9799	19.5986	0.1010	0.0510	14
15	0.4810	10.3797	63.2880	2.0789	21.5786	0.0963	0.0463	15

Example 1

what present sum will yield \$ 1000 in 5 years with interest 10%

Solution:

$P = 1000\,(1.1)^{-5} = \$620.92$

Depositing \$620.92 at 10% compounded annually will yield 1,000 in 5 years

Example 2

what interest rate is required to triple $1,000 in 10 years.

Solution:

$3000 = 1000\ (1+i)^{10}$

$i = 11.6\%$

Example 3

Given that a $40,000 pile jacketing will be required on a bridge in year 20 of its 50 year life, find the Present Worth of that expenditure (Interest 7%).

Solution:

Find P given F.

$P = 40{,}000[1/(1.07)^{20}] = \$10{,}337$ or

$P = 40{,}000 \times (P/F, 7\%, 20 \text{ yrs})$

$= 40{,}000 \times (0.2584) = \$10{,}336.$

Example 4

As a check on Example 1, fine the Future Worth in year 20 of an initial outlay of \$10,337 (Interest 7%).

Solution:

Find F given P.

$F = 10{,}337 \times (1 + 0.07)^{20} = \$40{,}001$ or

$F = 10{,}337 \times (F/P, 7\%, 20)$

$= 10{,}337 \times (3.8697) = \$40{,}001$

Example 5

A new roadway project costs \$2,100,000. What is the Annual Worth of this initial cost? Assume a 40 year life. (Interest 7%).

Solution:

Find A given P:

$A = 2{,}100{,}000\{[0.07(1.07)^{40}]/[1.07^{40} - 1]\} = \$157{,}519$ or

$A = 2{,}100{,}000 \times (A/P, 7\%, 40)$

$= 2{,}100{,}000 \times (0.0750) = \$157{,}500$

Example 6

As a check of Example 3, find the Present Worth of an annual outlay of $157,519. (Interest 7%).

Solution:

Find P given A.

$P = 157{,}519\{[(1.07)^{40} - 1]/[0.07(1.07)^{40}]\} = \$2{,}099{,}997$ or

$P = 157{,}519 \times (P/A, 7\%, 40)$

$= 157{,}519 \times (13.3317) = \$2{,}099{,}997$

Example 7

Find the Annual Worth of a $750,000 bridge widening project in year 50 of a bridge's life. (Interest 7%).

Solution:

Find A given F.

$A = 750{,}000\{(0.07)/[(1.07)^{50} - 1]\} = \$1{,}845$ or

$A = 750{,}000 \times (A/F, 7\%, 50)$

$= 750{,}000 \times (0.0025) = \$1{,}875$

Example 8

As a check on Example 5, find the Future Worth of an annual outlay of $1,845. (Interest 7%).

Solution:

Find F given A.

$F = 1,845[(1.07^{50} - 1)/(0.07)]$ or

$F = 1,845 \times (F/A, 7\%, 50$

$= 1,845 \times (406.5289) = \$750,046$

Engineering Economic Analysis Examples

Factor Name	Converts	Symbol	Formula
Single payment compound interest	P to F	(F/P,i%,n)	$(1 + i)^n$
Present Worth	F to P	(P/F,i%,n)	$1/(1 + i)^n$
Uniform series Sinking Fund	F to A	(A/F,i%,n)	$i/[(1 + i)^n - 1]$
Capital Recovery	P to A	(A/P,i%,n)	$[i(1 + i)^n]/[(1 + i)^n - 1]$
Compound amount	A to F	(F/A,i%,n)	$[(1 + i)^n - 1]/i$
Equal series present worth	A to P	P/A,i%,n)	$[(1 + i)^n - 1]/[i(1 + i)^n]$

Example 9

A construction company is comparing between 2 machines:

- The price of the first machine is 100,000 and will be sold after 5 years by 20,000
- The price of the other machine is 150,000 and will be sold after 5 years by 40,000

Which machine is more feasible to purchase? (i=10%)

Example 9

Solution

- PW [Machine (1)] = -100,000 + 20,000/(1+0.1)^5 = -87,581.5
- PW [Machine (2)] = -150,000 + 40,000/(1+0.1)^5 = -125,163.1

Machine 1 is better since its cost is less

Example 10

Two alternative plans are available for increasing the capacity of existing water transmission line. discount ratio =12%

	Plan A Pipeline	Plan B Pumping station
Construction cost	$1,000,000	$200,000
Life	40 years	40 years (structure) 20 years (equipment)
Operating cost	$1,000/year	$50,000/year
Cost of replacing equipment at the end of year 20	0	$75,000

Example 10

Solution:

Present Worth (Plan A) =

= P + A(P/A, 12%, 40) = \$1,000,000 + \$1000(8.24378)

= \$1,008,244

Present Worth (Plan B) =

= P + A(P/A, 12%, 40) + F(P/F, 12, 20%)

= \$200,000 + \$50,000(8.24378) + \$75,000(0.10367)

= \$619,964

Example 11

- The construction of a sewerage system is estimated to be $30,000,000.
- The annual operation, maintenance and repair (OMR) is $1,000,000/year.
- The annual income (benefit) from users is $3,500,000/year.
- The life of the system is 30 years and the discount rate is 5%.
- Determine if the project is *feasible or not*.

Example 11

Solution

- Annual Benefits = 3,500,000
- Annual OMR = -1,000,000
- Annual cost of construction = -30,000,000 (0.06505) = -1,951,500
- Net annual benefits (AW)= 3,500,000 - 1,000,000 - 1,951,500 = (+548,500)
- The positive means that the project is profitable

Example 12

Repeat Example 11 using the present Worth Method

Solution

- PW(Annual Benefits) = 3,500,000 x 15.3724 = 53,803,400
- PW(Annual OMR) = -1,000,000 x 15.3724 = -15,372,400
- PW(Annual cost of construction) = -30,000,000
- Net PW= 8,431,000

The positive means that the project is profitable

Alternatives with different life time

- Alternatives with unequal life times may be compared by assuming replacement at the end of the shorter life, thus maintaining the same level of uniform payment
- OR, all cash flows are changed to series of uniform payments

Example 13

- A company is investigating the installation of two alternative systems.
- Given the purchase price and the annual insurance and life, which system should be chosen, considering discount ratio =10%?

	System Cost	Insurance Premium	Life
Partial System	\$8,000	\$1,000	15 yr
Full system	\$15,000	\$250	20 yr

Example 13

Solution:

Annual cost (partial system) = A + P(A/P, 10%, 15)

= -\$1000 - \$8000(0.13147)

= -\$2051.75

Annual cost (Full system) = A + P(A/P, 10%, 20)

= -\$250 - \$15000(0.11746)

= -\$2011.90

The Full system is more economical

Example 14

Consider the relative of costs of a timber bridges and a steel one. Their initial cost and annual maintenance costs are given as follows. **Decide which bridge should be selected** considering discount ratio = 8%.

	Timber	Steel
Initial Cost	500,000 EGP	700,000 EGP
Annual Maintenance	30,000 EGP	5,000 EGP
life	15 years	30 years

Example 14

	Timber	Steel
Initial Cost	500,000 EGP	700,000 EGP
Annual Maintenance	30,000 EGP	5,000 EGP
life	15 years	30 years
Method (1) using present Worth Method (PW)		
PW of maintenance cost	30,000 (P/A,i,15) = 30,000 x 8.559 = 256,800 EGP	5,000 (P/A,i,30) = 5,000 x 11.258 = 56,290EGP
Present worth of renewal	500,000 (P/F,i,15) = 500,000 x 0.315 = 157,620.8 EGP	-
Present Worth of total costs	=500,000+ 256,8000+ 157,620.8 = **914,420.8E GP**	=700,000+ 56,290 = **756,290 EGP**
STEEL BRIDGE IS BETTER		

Example 14

	Timber	Steel
Initial Cost	500,000 EGP	700,000 EGP
Annual Maintenance	30,000 EGP	5,000 EGP
life	15 years	30 years
Method (2) using Annual Worth Method (AW)		
AW of Initial Cost	500,000 (A/P,i,15) = 500,000 x 0.1163 = 58,410EGP	700,000 (A/P,i,30) = 700,000 x 0.08882 = 62,174EGP
Total AW	= 30,000 + 58,410 = **88,410 EGP**	=62,174 + 5,000 = **67,174 EGP**
STEEL BRIDGE IS BETTER		

Example 15

A new piece of equipment costs **L.E.**100,000. The life of the equipment is estimated to be 15 years. During the first five years, there will be no maintenance cost. After that, L.E.20,000 is the annual maintenance cost. The equipment is assumed useless at the end of its life. Compute the equivalent annual cost of owing the machine by taking i=10%

Example 15

Solution:

- PW of the annual maintenance at year (5) =20,000 (P/A,10%,10)= 20,000 x 6.1455 = **L.E.**122,890
- PW of the annual maintenance at year (0) = 122,890 /$(1+0.1)^5$= 122,890 x 0.62092 = **L.E.**76,305
- Total PW = 100,000 + 76,305 = **L.E.**176,305
- Equivalent annual worth = 176,305 (A/P,10%, 15) = 176,305 x 0.1314 = **L.E.23,166.47**

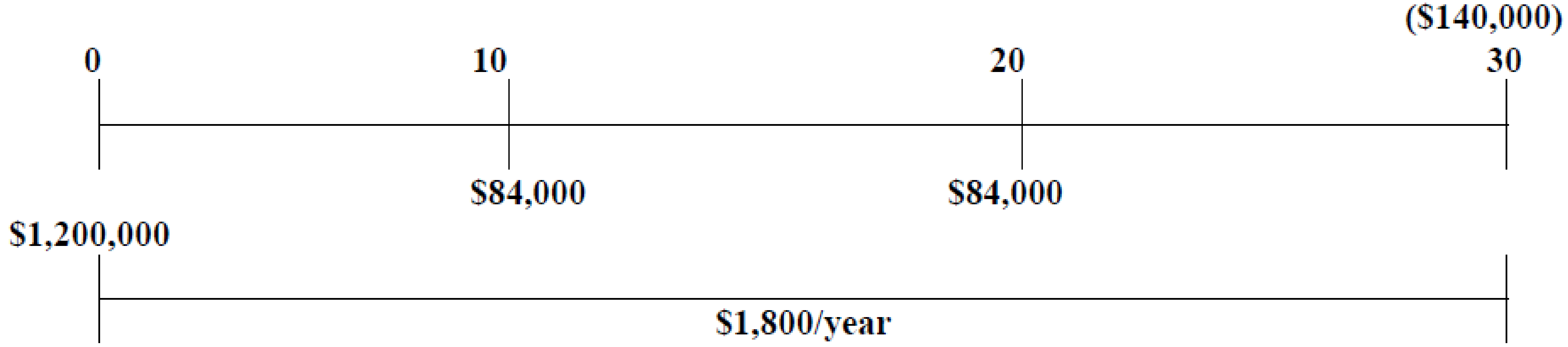
Example 16

	Alternative 1 PCC Pavement	Alternative 2 HMA Pavement
Initial Construction Cost (year 0)	$1,200,000	$900,000
Stage II Construction (year 10)		$350,000
Stage III Construction (year 20)		$290,000
Joint Sealing (year 10 & 20)	$84,000	
Routine Annual Maintenance	$1,800	$1,000
Salvage (year 30)	($140,000)	($280,000)

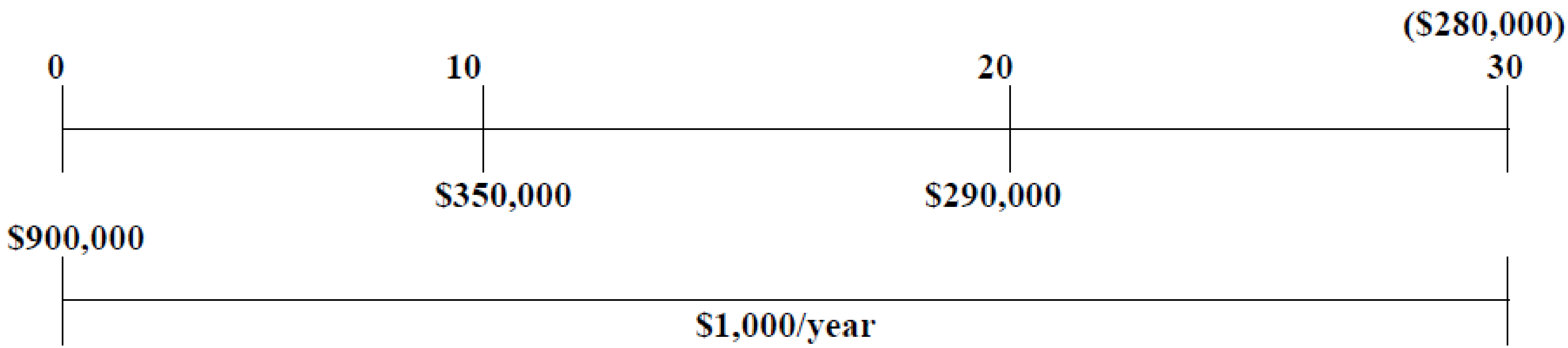
The estimated life of each alternative is 30 years.

Use a 4% discount rate to find the best alternative.

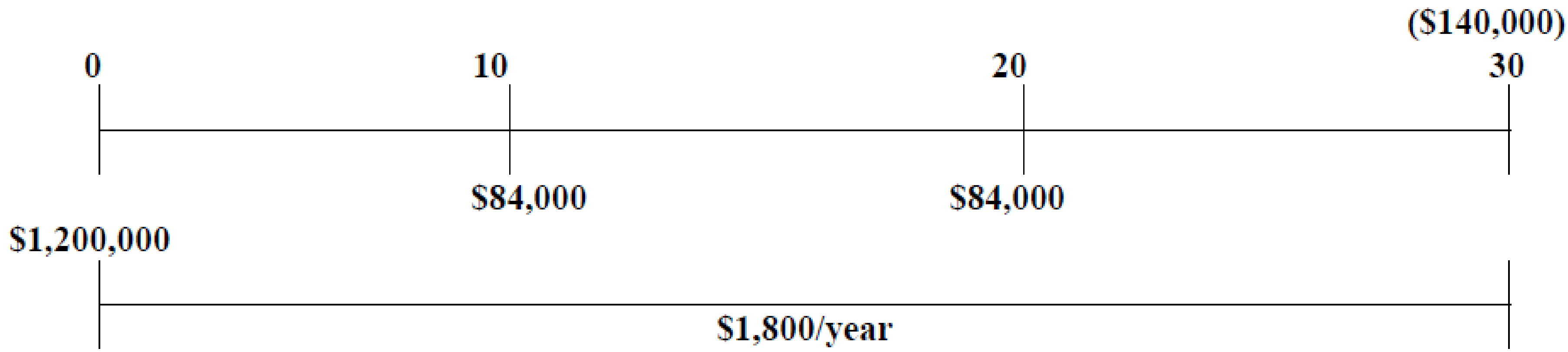
Alternative 1



Alternative 2



Alternative 1



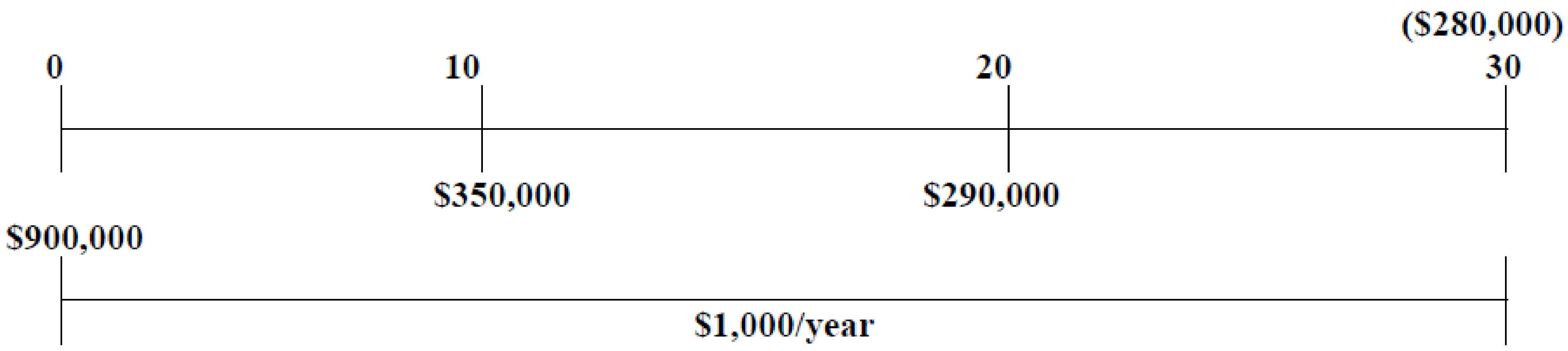
Present Worth Method

- P = $1,200,000 + $84,000 (P/F, 4%, 10) + $84,000 (P/F, 4%, 20) + $1,800 (P/A, 4%, 30) - $140,000 (P/F, 4%, 30)
- P = 1,200,000 + 84,000 (0.6756) + 84,000 (0.4564) + 1,800 (17.2920) - 140,000 (0.3083)= $1,283,045 ANSWER

Annual Worth Method

- A = $1,200,000 (A/P, 4%, 30) + $84,000 (P/F, 4%, 10) (A/P, 4%,30)+ $84,000 (P/F, 4%, 20) (A/P, 4%, 30) + $1,800 - $140, 000 (A/F, 4%, 30)
- A = 1,200,000 (0.0578) + 84,000 (0.6756) (0.0578) + 84,000 (0.4564) (0.0578)+ 1,800 - 140,000 (0.0178)= $74,199 ANSWER

Alternative 2



Present Worth Method

- P = $900,000 + $350,000 (P/F, 4%, 10) + $290,000 (P/F, 4%, 20) + $1,000 (P/A, 4%, 30) - $280,000 (P/F, 4%, 30)
- P = 900,000 + 350,000 (0.6756) + 290,000 (0.4564) + 1,000 (17.2920) - 280,000 (0.3083) = $1,199,762 ANSWER

Annual Worth Method

- A = $900,000 (A/P, 4%, 30) + $350,000 (P/F, 4%, 10) (A/P, 4%,30) + $290,000 (P/F, 4%, 20) (A/P, 4%, 30) + $1,000 - $280, 000 (A/F, 4%, 30)
- A = 900,000 (0.0578) + 350,000 (0.6756) (0.0578) + 290,000 (0.4564) (0.0578) + 1,000 - 280,000 (0.0178) = $69,382 ANSWER

Example 16

Comparison of Alternatives:

	Alternative 1	Alternative 2
Present Worth	$1,283,045	$1,199,762
Annual Worth	$74,199	$69,382

Alternative 2 is the least expensive alternative.

This example also illustrates that the use of either the annual worth or present worth method leads to the same conclusion.

Questions

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